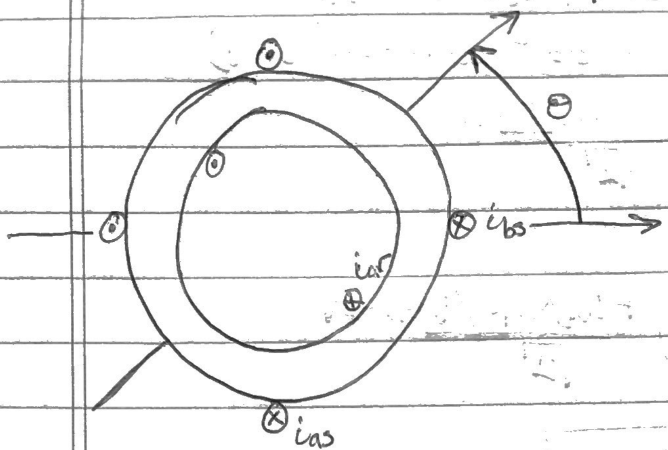
Frequency Condition: $\omega_m = \omega_s - \omega_r$

Synchronous machines: $\omega_m = \omega_s$

$\omega_r = 0$ (DC current for i_{ar}, $i_{br} = 0$)



$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{ar} \end{bmatrix} = \begin{bmatrix} L_s & 0 & M\cos(\theta) \\ 0 & L_s & M\sin(\theta) \\ M\cos(\theta) & M\sin(\theta) & L_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{ar} \end{bmatrix}$$

Assume: $i_{as} = I_s \cos(\omega_s t)$ $\qquad i_{ar} = I_r$

$i_{bs} = I_s \sin(\omega_s t)$ $\qquad \theta = \omega_m t + \gamma \Rightarrow \theta = \omega_s t + \gamma$

$$\lambda_{as} = L_s I_s \cos(\omega_s t) + M I_r \cos(\omega_s t + \gamma)$$
$$\lambda_{bs} = L_s I_s \sin(\omega_s t) + M I_r \sin(\omega_s t + \gamma)$$
$$\lambda_{ar} = M I_s \left(\cos(\omega_s t) + \sin(\omega_s t)\right) + L_r I_r$$

a phase Stator Voltage

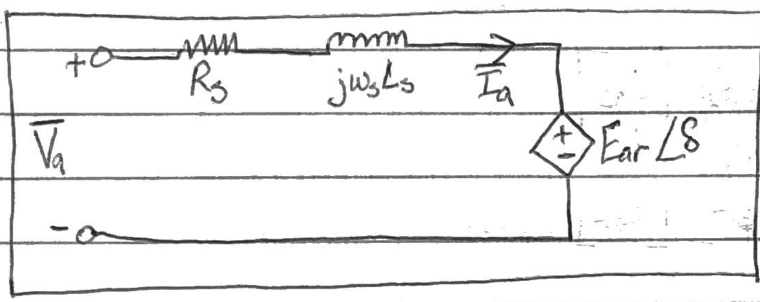
$$V_{as} = i_{as}R_s + \frac{d\lambda_{as}}{dt}$$

$$V_{as} = I_s R_s \cos(\omega_s t) + \left(-L_s I_s \omega_s \sin(\omega_s t) - M I_r \omega_s \sin(\omega_s t + \gamma)\right)$$

$$V_a \cos(\omega_s t + \theta_v) = I_s R_s \cos(\omega_s t) - \omega_s L_s I_s \sin(\omega_s t) - \omega_s M I_r \sin(\omega_s t + \gamma)$$

$$\Rightarrow \frac{V_a}{\sqrt{2}} \angle \theta_v = \frac{I_s R_s}{\sqrt{2}} \angle 0 + \frac{\omega_s L_s I_s}{\sqrt{2}} \angle 90^\circ + \frac{\omega_s M I_r}{\sqrt{2}} \angle \gamma + 90^\circ$$

$$\frac{V_a}{\sqrt{2}} \angle 0 = R_s \left(\frac{I_s}{\sqrt{2}} \angle -\theta_v\right) + j\omega_s L_s \left(\frac{I_s}{\sqrt{2}} \angle -\theta_v\right) + \frac{\omega_s M I_r}{\sqrt{2}} \angle \gamma + 90^\circ - \theta_v$$



$E_{ar} = \frac{\omega_s M I_r}{\sqrt{2}}$: excitation voltage, dependent on I_r

δ: torque angle

* Similar to per phase equivalent circuit from Chapter 2.
* Can use 1ϕ power to calculate 3ϕ power at Sychronous machine.

3 Phase Power: $P_{3\phi} = 3P_{1\phi} \Rightarrow P_{3\phi} = 3\,\mathrm{Re}\{\bar{E}_{ar}\bar{I}_a^*\}$

$$\bar{I}_a = \frac{\bar{V}_a - \bar{E}_{ar}}{R_s + jX_s}$$

$$P_{3\phi} = 3\,\mathrm{Re}\left\{ \bar{E}_{ar}\left(\frac{\bar{V}_a - \bar{E}_{ar}}{R_s + jX_s}\right)^* \right\} \Rightarrow P_{3\phi} = 3\,\mathrm{Re}\left\{ \frac{\bar{E}_{ar}\bar{V}_a^*}{R_s - jX_s} - \frac{E_{ar}^2}{R_s - jX_s} \right\}$$

$$P_{3\phi} = 3\,\mathrm{Re}\left\{ \frac{\bar{E}_{ar}\bar{V}_a^*}{R_s\left(1 - j\frac{X_s}{R_s}\right)} - \frac{E_{ar}^2}{R_s\left(1 - j\frac{X_s}{R_s}\right)} \right\} \Rightarrow P_{3\phi} = 3\,\mathrm{Re}\left\{ \frac{\bar{E}_{ar}\bar{V}_a^*}{X_s\left(\frac{R_s}{X_s} - j\right)} - \frac{E_{ar}^2}{X_s\left(\frac{R_s}{X_s} - j\right)} \right\}$$

if $X_s \gg R_s$: $$P_{3\phi} = 3\,\mathrm{Re}\left\{ \frac{\bar{E}_{ar}\bar{V}_a^*}{-jX_s} - \frac{E_{ar}^2}{-jX_s} \right\} \Rightarrow P_{3\phi} = 3\,\mathrm{Re}\left\{ \frac{j\bar{E}_{ar}\bar{V}_a^*}{X_s} - j\frac{E_{ar}^2}{X_s} \right\}$$

$$P_{3\phi} = 3\,\mathrm{Re}\left\{ j\frac{\bar{E}_{ar}\bar{V}_a^*}{X_s} \right\} \Rightarrow P_{3\phi} = 3\,\mathrm{Re}\left\{ \frac{E_{ar}\angle\delta\,(V_a\angle 0)\,(1\angle 90^\circ)}{X_s} \right\}$$

$$P_{3\phi} = 3\,\mathrm{Re}\left\{ \frac{E_{ar}V_a\angle \delta + 90^\circ}{X_s} \right\}$$

$$\boxed{P_{3\phi} = -\frac{3E_{ar}V_a\sin(\delta)}{X_s}}$$

* This is power consumed by the synchronous machine.

$$P_m = T^e\omega_m \Rightarrow P_m = T^e\omega_s \Rightarrow \quad \boxed{T^e = \frac{P_{3\phi}}{\omega_s}}$$

Motor: $\delta < 0$

$$P_{3\phi} = \frac{3E_{ar}V_a}{X_s}\sin(\delta)$$

Generator: $\delta > 0$

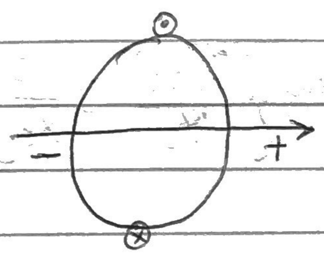
$$P_{3\phi} = -\frac{3E_{ar}V_a}{X_s}\sin(\delta)$$

$$\bar{S}_{3\phi} = 3\bar{V}_a \bar{I}_a^*$$
$$\bar{S}_{3\phi} = P_{3\phi} + jQ_{3\phi}$$

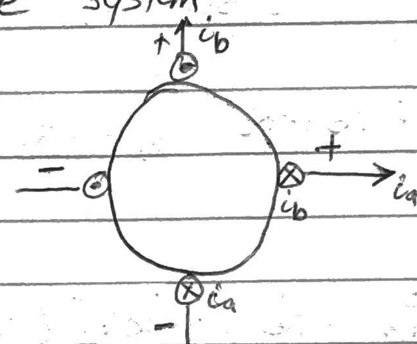
if $Q_{3\phi} < 0$: overexcited (supply VARs)
if $Q_{3\phi} > 0$: underexcited (consume VARs)

Multiple Poles

* Everything up to now was a 2 pole system



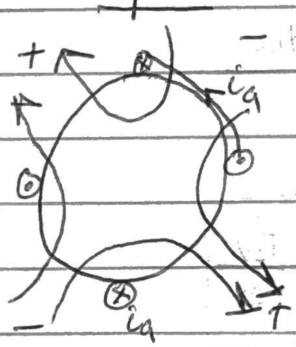
2 poles: 1 + and 1 −



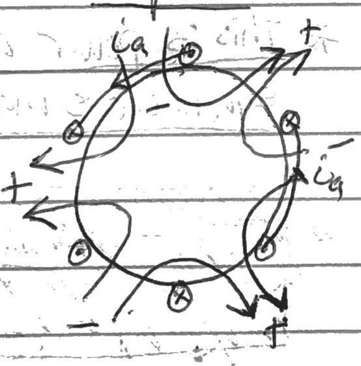
2 poles: 1 + and 1 − per phase

* The number of poles are the total + and − per phase

4 pole



6 pole



Changes to Equations for multi-pole systems

$p = \#$ of poles

$\theta_e = \left(\frac{p}{2}\right)\theta_m$

$\omega_e = \left(\frac{p}{2}\right)\omega_m$

* Don't have to travel as far in θ to switch current direction

θ_e: electrical angle

θ_m: mechanical angle

$\omega_m = \frac{2\pi}{60}(RPM)$

$\omega_e = 2\pi f$

$f = 60Hz \Rightarrow \omega_e = 120\pi$

$120\pi = \frac{2\pi}{60}(RPM)\left(\frac{p}{2}\right) \Rightarrow \boxed{RPM = \left(\frac{2}{p}\right)3600}$

p	RPM
2	3600
4	1800
6	1200
8	900
10	720

$P = T^e \omega_m$

$P = T^e\left(\frac{2}{p}\right)\omega_e$

$$\boxed{T^e = \frac{P}{\left(\frac{2}{p}\right)\omega_e}}$$

Ex 6.1 in textbook

3ϕ, 60Hz, $V_L = 550$ V, 6 pole, wye connected generator

$P_{3\phi} = 100$ kW generated

$E_{ar} = 460$ V/phase

$X_s = 2\Omega$

Find: a) δ and T^e

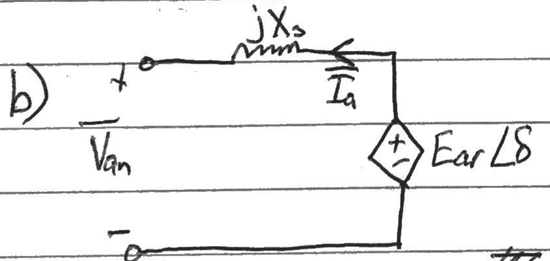
b) $\bar{I}_a$, PF, Q generated

Solution: a) $P_{3\phi} = \dfrac{3 E_{ar} V_{an} \sin(\delta)}{X_s} \Rightarrow \sin(\delta) = \dfrac{X_s P_{3\phi}}{3 E_{ar} V_{an}}$

$\sin(\delta) = \dfrac{2\Omega(100 \times 10^3 \text{ W})}{3_{\text{phase}}(460 \text{ V/phase})\left(\frac{550\text{V}}{\sqrt{3}}\right)} \Rightarrow \sin(\delta) = 0.4564$

$\boxed{\delta = 27.16^\circ}$

$T^e = \dfrac{P_{3\phi}}{\frac{2}{p}\omega_e} \Rightarrow T^e = \dfrac{100 \times 10^3 \text{ W}}{\left(\frac{2}{6}\right) 60\text{Hz}(2\pi)} \Rightarrow \boxed{T^e = 795.77 \text{ Nm}}$

b) 

$E_{ar}\angle\delta = jX_s \bar{I}_a + V_{an}\angle 0$

$\bar{I}_a = \dfrac{E_{ar}\angle\delta - V_{an}\angle 0}{jX_s}$

$\boxed{\bar{I}_a = 114.57\angle -23.60^\circ \text{ A}}$

$PF = \cos(23.60^\circ) \Rightarrow \boxed{PF = 0.916 \text{ lagging}}$

$S_{3\phi} = \dfrac{P_{3\phi}}{PF}$ $\quad Q = S_{3\phi} \sin(\theta) \Rightarrow \boxed{Q = 43.706 \text{ kVAR}}$